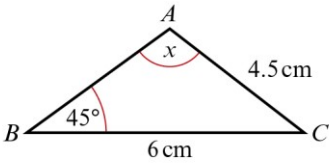
Exercise 6C

1 a



$x > 45°$

So there are two possible results.

Using $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\dfrac{\sin x}{6} = \dfrac{\sin 45°}{4.5}\sqrt{a^2+b^2}$

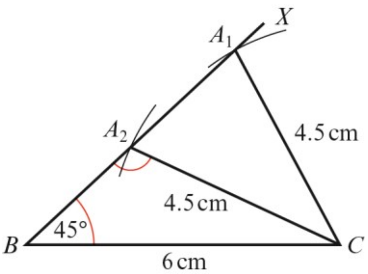
$\sin x = \dfrac{6\sin 45°}{4.5}$

$x = \sin^{-1}\left(\dfrac{6\sin 45°}{4.5}\right)$ or

$x = 180° - \sin^{-1}\left(\dfrac{6\sin 45°}{4.5}\right)$

$x = 70.5°\,(3\text{ s.f.})$ or $x = 109.5°\,(3\text{ s.f.})$

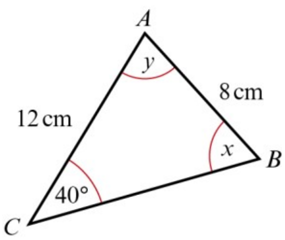
b



Draw BC = 6 cm.

Construct or draw an angle of 45° at B and extend the line as (BX).
Set the compasses to a radius of 4.5 cm.
Put the point on C and draw an arc.
The points where the arc meets BX are the two possible positions of A.

2 a



2 a Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin x}{12} = \dfrac{\sin 40°}{8}$

$\sin x = \dfrac{12\sin 40°}{8}$

$x = \sin^{-1}\left(\dfrac{12\sin 40°}{8}\right)$ or

$x = 180° - \sin^{-1}\left(\dfrac{12\sin 40°}{8}\right)$

$x = 74.6°$ or $x = 105.4°\ (3\text{ s.f.})$

When $x = 74.6°$:

$y = 180° - (74.6 + 40)°$

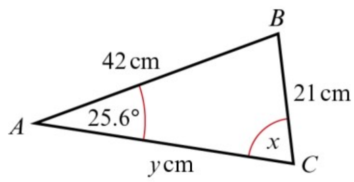
$= 65.4°\ (3\text{ s.f.})$

When $x = 105.4°$:

$y = 180° - (105.4 + 40)°$

$= 34.6°\ (3\text{ s.f.})$

b



Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin x}{42} = \dfrac{\sin 25.6°}{21}$

$\sin x = \dfrac{42\sin 25.6°}{21}$

$x = \sin^{-1}(2\sin 25.6°)$ or

$x = 180° - \sin^{-1}(2\sin 25.6°)$

$x = 59.8°$ or $x = 120°\ (3\text{ s.f.})$

When $x = 59.8°$:

angle $B = 180° - (59.8° + 25.6°) = 94.6°$

When $x = 120°$:

angle $B = 180° - (120.2° + 25.6°) = 34.2°$

2 b Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 94.6°} = \frac{21}{\sin 25.6°}$$

$$\text{So } y = \frac{21\sin 94.6°}{\sin 25.6°}$$

$$= 48.4 \text{ cm (3 s.f.)}$$

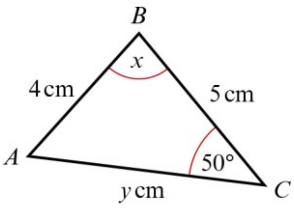
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 34.2°} = \frac{21}{\sin 25.6°}$$

$$\text{So } y = \frac{21\sin 34.2°}{\sin 25.6°}$$

$$= 27.3 \text{ cm (3 s.f.)}$$

c



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{5} = \frac{\sin 50°}{4}$$

$$\sin A = \frac{5\sin 50°}{4}$$

$$A = \sin^{-1}\left(\frac{5\sin 50°}{4}\right) \text{ or}$$

$$A = 180° - \sin^{-1}\left(\frac{5\sin 50°}{4}\right)$$

$$A = 73.25° \text{ or } A = 106.75°$$

When $A = 73.247°$:

$$x = 180° - (50 + 73.247)°$$

$$= 56.8° \text{ (3 s.f.)}$$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{y}{\sin x} = \frac{4}{\sin 50°}$$

$$\text{So } y = \frac{4\sin x}{\sin 50°}$$

$$= 4.37 \text{ cm (3 s.f.)}$$

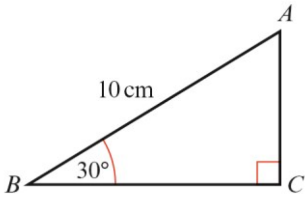
2 c When $A = 106.75°$:

$$x = 180° - (50 + 106.75)° = 23.2° \text{ (3 s.f.)}$$

As above:

$$y = \frac{4\sin x}{\sin 50°} = 2.06 \text{ cm (3 s.f.)}$$

3 a



The length of AC is least when it is at right angles to BC.

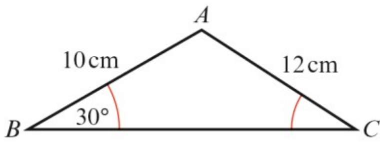
Using $\sin B = \frac{AC}{AB}$

$$\sin 30° = \frac{AC}{10}$$

$$AC = 10\sin 30° = 5$$

$$AC = 5 \text{ cm}$$

b



Using $\frac{\sin C}{c} = \frac{\sin B}{b}$

$$\frac{\sin C}{10} = \frac{\sin 30°}{12}$$

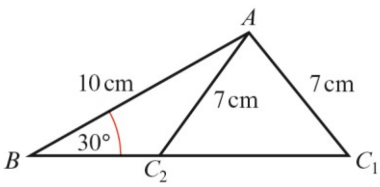
$$\sin C = \frac{10\sin 30°}{12}$$

$$C = \sin^{-1}\left(\frac{10\sin 30°}{12}\right)$$

$$= 24.62°$$

$$\angle ABC = 24.6° \text{ (3 s.f.)}$$

c

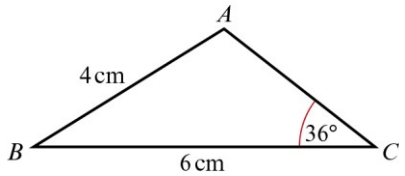


As 7 cm < 10 cm, $\angle ACB > 30°$.

3 c There are two possible results.
Using 7 cm instead of 12 cm in **b**:

$$\sin C = \frac{10\sin 30°}{7}$$
$$C = \sin^{-1}\left(\frac{10\sin 30°}{7}\right) \text{ or}$$
$$C = 180° - \sin^{-1}\left(\frac{10\sin 30°}{7}\right)$$
$$C = 45.58° \text{ or } 134.4°$$
$$\angle ABC = 45.6° (3 \text{ s.f.}) \text{ or } 134° (3 \text{ s.f.})$$

4

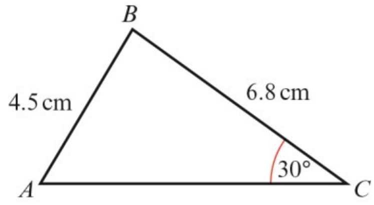


As $4 < 6$, $36° < \angle BAC$, so there are two possible values for angle A.

$$\text{Using } \frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin A}{6} = \frac{\sin 36°}{4}$$
$$\sin A = \frac{6\sin 36°}{4}$$
$$A = \sin^{-1}\left(\frac{6\sin 36°}{4}\right) \text{ or}$$
$$A = 180° - \sin^{-1}\left(\frac{6\sin 36°}{4}\right)$$
$$A = 61.845\ldots \qquad \ldots$$
When $A = 118.154\ldots$
$$\angle ABC = 180° - (36° + 118.154\ldots$$
$$= 25.8° \ (3 \text{ s.f.})$$

Using this value for $\angle ABC$ with
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{AC}{\sin 25.8°} = \frac{4}{\sin 36°}$$
$$\text{So } AC = \frac{4\sin 25.8°}{\sin 36°}$$
$$= 2.96 \text{ cm } (3 \text{ s.f.})$$

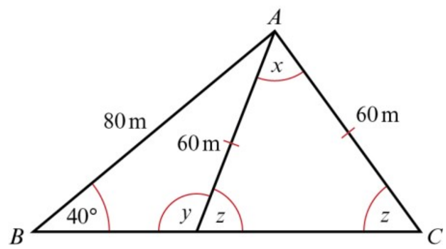
5



As $6.8 > 4.5$, angle $A > 30°$ and so there are two possible values for A.

$$\text{Using } \frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin A}{6.8} = \frac{\sin 30°}{4.5}$$
$$A = \sin^{-1}\left(\frac{6.8\sin 30°}{4.5}\right) \text{ or}$$
$$A = 180° - \sin^{-1}\left(\frac{6.8\sin 30°}{4.5}\right)$$
$$A = 49.07\ldots \qquad \ldots$$
When $A = 49.07\ldots°$, B is the largest angle.
$$\angle ABC = 180° - (30° + 49.07\ldots$$
$$= 101° \ (3 \text{ s.f.})$$
When $A = 130.926\ldots°$, this the largest angle.
$$\angle BAC = 131° (3 \text{ s.f.})$$

6 a



Using the sine rule:
$$\frac{\sin y}{80} = \frac{\sin 40°}{60}$$
$$\sin y = \frac{80\sin 40°}{60}$$
$y = 59°$ or $121°$
y is obtuse, therefore, $y = 121°$
$z = 59°$
$x = 180° - 2 \times 59° = 62°$
$x = 62°$

b The assumption is that the ball swings symmetrically.